Find the Maclaurin series for f(x) using the definition of a Maclaurin series. Assume that f has a power series expansion. Do not show that $R_n(x) \rightarrow 0$. Also find the associated radius of convergence.

1)
$$f(x) = \cos x$$
 $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad R = \infty$

2)
$$f(x) = \sin 2x$$
 $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}, \quad R = \infty$

3)
$$f(x) = (1+x)^{-3}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)(n+1)x^n}{2}, \quad R = 1$$

4)
$$f(x) = xe^x$$
 $\sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}, \quad R = \infty$

Find the Taylor series for f(x) centered at the given value of a. Assume that f has a power series expansion. Do not show that $R_n(x) \rightarrow 0$. Also find the associated radius of convergence.

5)
$$f(x) = 1 + x + x^2$$
, $a = 2$ $7 + 5(x-2) + (x-2)^2$, $R = \infty$

6)
$$f(x) = e^x$$
, $a = 3$ $\sum_{n=0}^{\infty} \frac{e^3}{n!} (x-3)^n$, $R = \infty$

7)
$$f(x) = \sin x$$
, $a = \frac{\pi}{2}$ $\sum_{n=0}^{\infty} (-1)^n \frac{(x - \pi/2)^{2n}}{(2n)!}$, $R = \infty$

8)
$$f(x) = \frac{1}{\sqrt{x}}, \quad a = 9$$
 $\sum_{n=0}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2^n \cdot 3^{2n+1} \cdot n!} (x-9)^n, \quad R = 9$

Use a derived Maclaurin series to obtain the Maclaurin series for the given function. Also find the associated radius of convergence.

9)
$$f(x) = e^{-x/2}$$
 $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^n, \quad R = \infty$

10)
$$f(x) = x \tan^{-1} x$$
 $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+2}}{2n+1}$, $R = 1$

11)
$$f(x) = x \cos 2x$$
 $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} x^{2n+1}, \quad R = \infty$

12)
$$f(x) = \sin^2 x$$
 [Hint: Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.] $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$, $R = \infty$

Evaluate the indefinite integral as an infinite series.

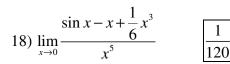
14) $\int x \cos(x^3) dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!}$

15)
$$\int \frac{\sin x}{x} dx$$
 $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$

16)
$$\int \frac{e^x - 1}{x} dx \qquad \qquad C + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$$

Use series to evaluate the limit.

17)
$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$$
 $\frac{1}{3}$



Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for each function.

19)
$$y = e^{-x^2} \cos x$$
 $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4 + \cdots$

20)
$$y = \frac{x}{\sin x}$$
 $1 + \frac{1}{6}x^2 + \frac{7}{360}x^4 + \cdots$

Find the sum of the series.

21)
$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$$
 e^{-x^4}

22)
$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!}$$
 $\overline{\frac{\sqrt{3}}{2}}$

23)
$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots \qquad \boxed{\frac{1}{2}}$$